Exercise 156

For the following exercises, solve the trigonometric equations on the interval $0 \leq \theta < 2\pi$.

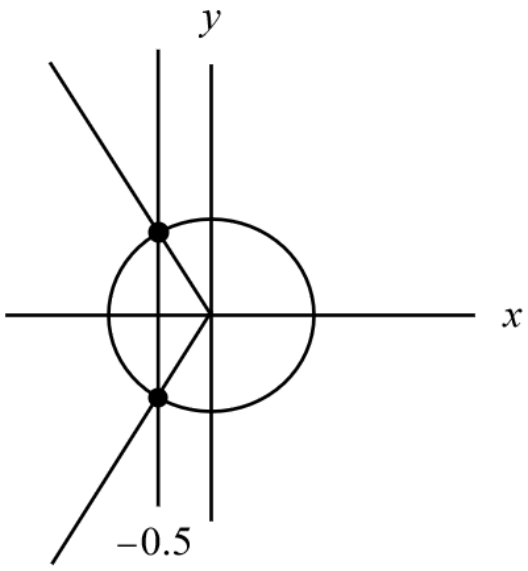
$$1 + \cos\theta = \frac{1}{2}$$

Solution

$$1 + \cos\theta = \frac{1}{2}$$

$$\cos\theta = -\frac{1}{2}$$

We want the two angles to the points on the unit circle that are a distance $1/2$ to the left.



Taking the inverse cosine of $-1/2$ gives 120°, or $2\pi/3$ radians. This is the counterclockwise angle from the positive x-axis to the point on top in the figure. The angle to the point on the bottom is the same but negative, −120°, or $-2\pi/3$ radians. Add 2π to it so that it's between 0 and 2π.

$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}$$

Therefore,

$$\theta = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}.$$